Bilinear Algebra and Geometry

1. **Course number and name:** 020ALBNI3 Bilinear Algebra and Geometry

2. **Credits and contact hours:** 6 ECTS credits, 3x1:15 contact hours

3. **Name(s) of instructor(s) or course coordinator(s):** Rim Aldbaissy, Joanna Abou Nader, Fida El Chami, Rana Fakhreddine, Lara Saliba, Diala Wehbe.

4. **Instructional materials:** PowerPoint slides; lecture notes; worksheets.

   Reference:
   – Maths PTSI, X. Oudot et V. Queffelec, Vuibert, 2014.

5. **Specific course information**
   a. **Catalog description:**
      This course provides students with a solid understanding of fundamental concepts, including the reduction of endomorphisms, pre-Hilbert spaces and endomorphisms of Euclidean spaces. Throughout this course, students will develop proficiency in techniques for reducing matrices and endomorphisms, along with their practical applications such as calculating matrix powers, solving linear recurrent sequence systems and utilizing linear recurrent sequences for matrix exponential. Additionally, the course examines pre-Hilbert spaces, placing emphasis on key notions such as the inner product, orthogonality and orthogonal projections. Students will learn applying these concepts in solving problems related to orthonormalization. Furthermore, the course covers the study of planar isometries, encompassing translations, rotations and reflections, as well as isometries in space. By engaging with these topics, students will acquire a strong foundation in bilinear algebra and the necessary skills to apply these concepts effectively in practical situations.

   b. **Prerequisites:** 020ALNNI2 Linear Algebra

   c. **Required /Selected Elective/Open Elective:** Required

6. **Educational objectives for the course**
   a. **Specific outcomes of instruction:**
      – Understand the concept of endomorphism reduction and its applications.
      – Develop the ability to identify eigenspaces and eigenvalues associated with an endomorphism.
      – Gain proficiency in diagonalizing and triangularizing matrices and endomorphisms.
      – Apply the techniques of endomorphism reduction to solve mathematical problems.
      – Acquire a comprehensive understanding of the structure and properties of pre-Hilbert spaces.

- Study the concepts of inner product, norm and orthogonality within the context of pre-Hilbert spaces.
- Know how to apply the Cauchy-Schwarz and triangle inequalities in pre-Hilbert spaces.
- Solve mathematical problems using the properties of pre-Hilbert spaces?
- Understand the properties and characteristics of endomorphisms in Euclidean spaces.
- Study the concepts of symmetry, orthogonality and isometry in endomorphisms of Euclidean spaces.
- Apply the properties of endomorphisms in Euclidean spaces to solve mathematical problems, including diagonalization and determination of eigenvalues.

b. PI addressed by the course:

PI	1.3	7.1
Covered	x	x
Assessed	x	

7. **Brief list of topics to be covered**
   - Eigenvectors of an endomorphism (3 lectures)
   - Characteristic polynomial (2 lectures)
   - Diagonalization (3 lectures)
   - Polynomial of an endomorphism: Stability of a subspace by an endomorphism, Polynomial of an endomorphism, Minimal polynomial of an endomorphism (3 lectures)
   - Triangularization, Triangularizable endomorphisms, Characteristic subspaces (3 lectures)
   - Application of reduction (3 lectures)
   - Inner product and associated norm: Inner product, Norm associated with an inner product (3 lectures)
   - Orthogonality, Euclidean space (3 lectures)
   - Projections and orthogonal symmetries: Orthogonal complement of a set, Orthogonal complement of a finite-dimensional subspace, Orthogonal projection onto a finite-dimensional subspace (2 lectures)
   - Orthogonal symmetry with respect to a finite-dimensional subspace, distance to a subspace, Linear forms on a Euclidean space (2 lectures)
   - Orthogonal group (3 lectures)
   - Symmetric endomorphisms (3 lectures)
   - Euclidean planes (1 lectures)
   - Three-dimensional Euclidean spaces (2 lectures)