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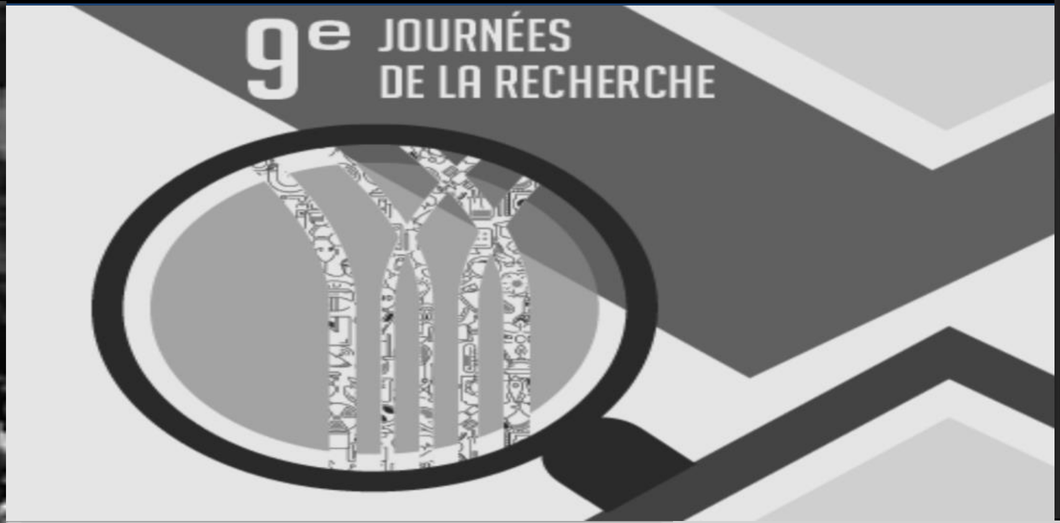

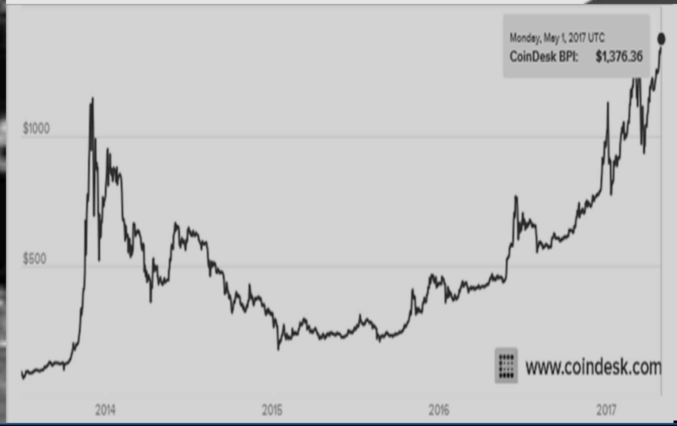

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Commodity Volatility Shocks And BRIC Sovereign Risk: A GARCH-Quantile Approach

Dr. Naji Pierre Jalkh | Saint-Joseph University

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Commodity volatility shocks and BRIC sovereign risk: A GARCH-quantile approach

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Agenda

Commodity Volatility Shocks And BRIC Sovereign Risk: A GARCH-Quantile Approach

Commodity Volatility Shocks And BRIC Sovereign Risk: A GARCH-Quantile Approach

- Motivation Behind The Study and Literature Review
- The Research Methodology
- Data Presentation and Discussion
- Empirical Results

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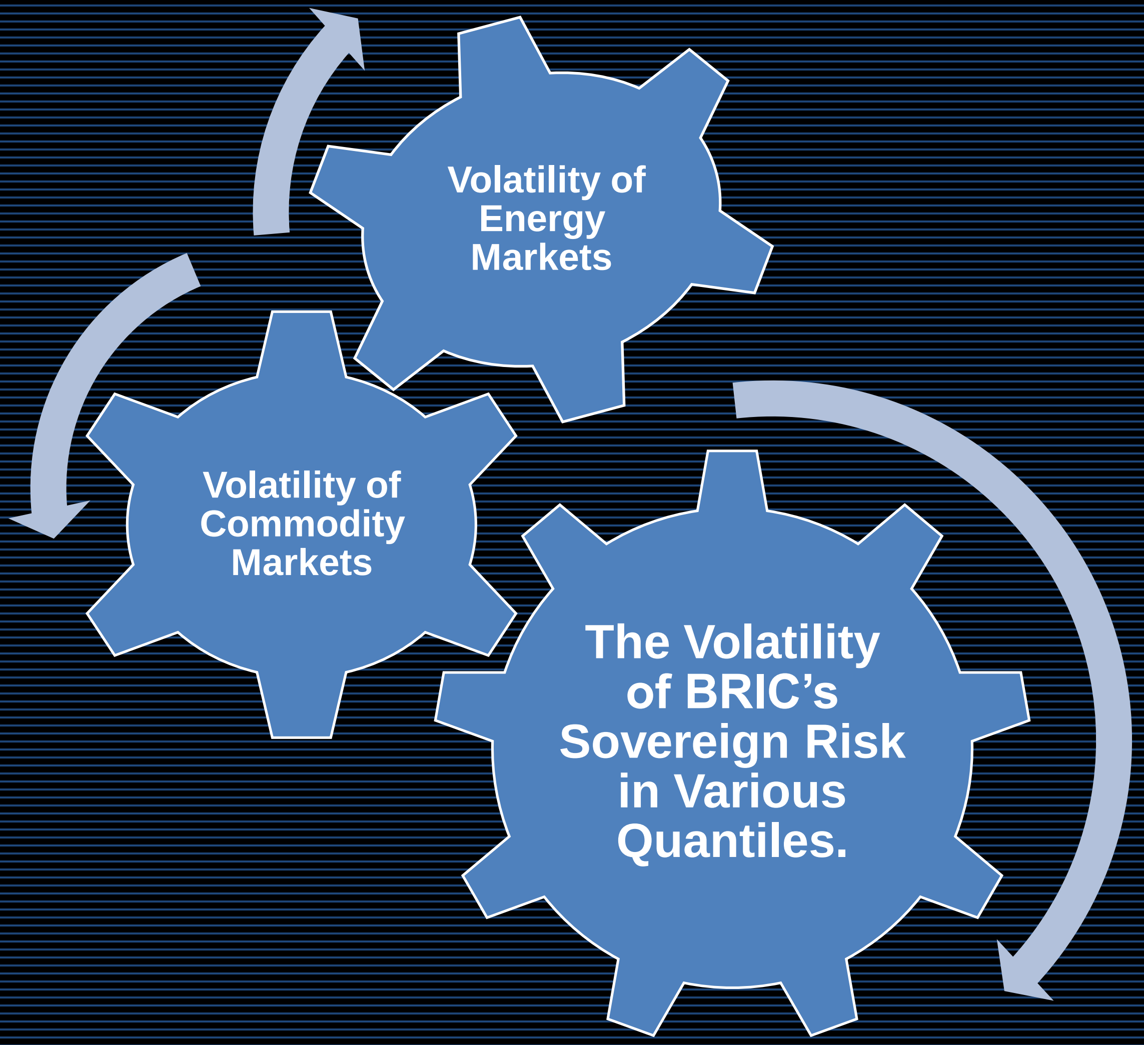
Motivation Behind The Study and Literature Review

Commodity Volatility Shocks And BRIC Sovereign Risk: A GARCH-Quantile Approach

Motivation Behind The Study and Literature Review

- *We study whether the contemporaneous and lagged volatility of the commodity / energy markets can help predict the volatility of Brazil, Russia, India, China (BRIC) sovereign risk in the quantiles, i.e. under low, moderate, and high volatility conditions.*

- *We employ a quantile-based approach to uncover the impact of quantile (energy) commodity price volatility on different volatility of sovereign CDS spreads quantiles, while accounting for the effect of the mid-2014 energy price decline.*



Motivation Behind The Study and Literature Review

Unlike advanced economies *(e.g. Norway, Australia and Canada)*, most of *emerging economies*

- (a) are less diversified,
- (b) have lower credit rating, and
- (c) still depend on commodities and energy exportations (UNDP).

Bhar and Nikolova (2009) argue that *emerging economies*

- are more vulnerable to energy price volatility than developed countries because they grow more rapidly and have more energy intensive production structures.
- Importantly, the price volatility of commodity and energy leads to macroeconomic instabilities
- as well as to a volatility in: (a) export earnings, (b) foreign exchange reserves, (c) current account balances, (d) economic activities,
- and eventually to (e) difficulties in meeting debt obligations.

Motivation Behind The Study and Literature Review

Hilscher and Nosbusch (2010)	argue that a country with historically higher macroeconomic volatility is *more prone to* ***default***.
Hooper (2015)	examines the association between *oil and gas reserves* and sovereign spreads in 10 emerging oil-exporting countries from 1994 to 2014.
	The author shows that *oil reserves* have an effect on sovereign spreads but this effect also depends on the institutional quality of the country, namely corruption, political stability, and democracy.

Motivation Behind The Study and Literature Review

However, the impact of *volatility* of (energy) commodities on the *volatility* of sovereign CDS spreads in large *emerging countries* remain extremely understudied

although numerous press articles have focused on the negative impact of the sharp energy prices decline since mid-2014 on the sovereign risk of emerging energy-exporting countries.

Lower Energy Prices

deteriorated fiscal balances in Russia that is strained to fund its public budget and pay off its external dollar-denominated debt.

Motivation Behind The Study and Literature Review

Sharma and Thuraisamy (2013)	highlight the role of *oil price uncertainty* in predicting sovereign CDS returns in eight Asian countries but don't account for the effect of this role on the volatility of CDS spreads.
Liu et al. (2016)	focus on the statistical properties of *country risk rating* in oil-exporting countries and show that *oil* price volatility can accentuate the volatility of *country risk ratings*.
Lee et al. (2017)	using a structural VAR approach to monthly data, show that country risk in net oil-importing (Germany, France, Italy, Japan and US) and net oil-exporting countries (Canada, UK) is affected by *oil price shocks*.

Motivation Behind The Study and Literature Review

An interesting study by Bouri et al. (2017)

focuses on the ***volatility spillover*** between commodities and sovereign CDS spreads of emerging and frontier economies and reports significant volatility linkages but that were found not be closely related to the levels of commodities/energy dependence in such economies.

However, Bouri et al. (2017) just concentrate on the average dependence in the volatility without taking into account the difference in the dependence between high and low quantiles. It is documented that the return and volatility distribution of assets is most often heterogeneous, suggesting the appropriateness of using a quantile approach to uncover differences in the response of the *volatility of sovereign CDS spreads* to changes in the *volatility of commodities* across the different quantiles.

Measuring how co-movement and Granger causality between the volatility of commodity/energy prices and the volatility of sovereign CDS spreads have changed under different volatility conditions (various quantiles) has implications for policy-makers as well as on investors in terms of risk management and predictability of sovereign CDS spreads.

Motivation Behind The Study and Literature Review

Why Consider Brazil, Russia, India, China (BRIC) Countries?

BRIC is a suitable and heterogeneous group of countries that includes major energy exporters and energy importers.

BRIC countries also attract a large portion of capital inflows (Net FDI $9.6 Billion, World Bank) and represent a major investment destination for global portfolio managers.

Why CDS Spread?

Often the decision of global investors to get into or out of emerging economies depends among others on *sovereign risk*, as measured by the sovereign CDS spread.

The latter is a risk measure closely watched by risk managers and policy-makers as it represents the cost at which a country can raise fund in international markets. For example, higher sovereign CDS spreads suggest that a country may face constraints in raising funds abroad and in attracting foreign investments.

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The Research Methodology

Commodity Volatility Shocks And BRIC Sovereign Risk: A GARCH-Quantile Approach

The Research Methodology

A Two-step Approach

The First Stage

- ***Best fitting of mean and variance equations for standard and GJR-ARMA-GARCH to all series.***

The Second Stage

- ***Extracted conditional volatility (GARCH) series are used within a quantile regression approach to capture the contemporaneous and lagged volatility transmission between commodity/energy prices and CDS spread changes of BRIC countries.***

The Research Methodology

The First Stage

- *Unlike the standard GARCH model of Bollerslev (1986), the asymmetric GARCH of Glosten et al. (1993), a.k.a.* ***GJR-GARCH model, adds an asymmetric term to capture the asymmetric response of the conditional variance to shocks****.*
- ***To decide which GARCH-based model has a superior fit and specifications****, i.e. symmetric (GARCH) or asymmetric (GJR-GARCH); and the density of the error distribution, normal, t-student, or generalized error distributions),* ***we follow Beine and Laurent(2003) and use SIC.***

The Research Methodology

The First Stage: A Closer Look

- ***The standard GARCH and the asymmetric GJR-GARCH model are given respectively in Eq. (1) and Eq. (2), respectively.***
  - $\boldsymbol{\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2}$ ***(1)***
- where σ_t^2 is the conditional variance, ε_{t-1} is the innovation, α represents the ARCH term which measures the impact of past innovations on current variance, and β represents the GARCH term which measures the impact of past variance on current variance.

- $\boldsymbol{\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma\varepsilon_{t-1}^2 I_\varepsilon\,(\varepsilon_{t-1})}$ ***(2)***
- where I is a dummy variable that measures the asymmetric response of the conditional variance to shocks. *It takes a value of unity in response to negative shocks and zero in response to positive shocks*.
- In addition to the constraints required by the sGARCH to ensure stationarity and positivity (ω>0; α ≥ 0; β ≥ 0; and α + β<1), an additional constraint has to be respected in the case of the GJR-GARCH (α +β +0.5γ<1).

The Research Methodology

A Two-step Approach

The First Stage

- ***Best fitting of mean and variance equations for standard and GJR-GARCH to the all series.***

The Second Stage

- ***Extracted conditional volatility (GJR-GARCH) series are used within a quantile regression approach to capture the contemporaneous and lagged volatility transmission between commodity/energy prices and CDS spread changes of BRIC countries.***

The Research Methodology (Cont.)

The Second Stage: Quantile Regression Based Analysis

- a very precise tool to model the *impact of the* ***conditional variables*** on the *dependent variable* (Koenker, 2005).
- identify the relation between the volatility of commodity/energy and that of sovereign CDS spreads ***under different market conditions.***

The Research Methodology

The Second Stage: A Closer Look

- We consider that the conditional τ –quantile of the volatility of CDS spreads distribution is affected by the contemporaneous and lagged volatility of the commodity and energy markets, while accounting for the impact of the mid-2014 energy price decline.

$$Q_{VCDS_t}(\tau|VCDS_{t-1}, d_t, X_t, X_{t-1}) = \alpha(\tau) + \beta(\tau)VCDS_{t-1} + \delta(\tau)d_t + \phi(\tau)X_t + \varphi(\tau)X_{t-1} \quad (3)$$

- where $VCDS_t$ represents the volatility of CDS spreads of a country; $VCDS_{t-1}$ is the lagged variable of the dependent variable; d_t is the dummy variable identifying the period before and after the mid-2014 energy decline denoted as 0 and 1, respectively; X_t represents a k × 1 vector of regressors, which is the constant and the energy (commodity) price index.

- ***The pair bootstrap method of Buchinsky (1995) is used to obtain the standards errors for the estimated coefficients.***

- *This method is used because it provides asymptotically valid standard errors under misspecifications of the quantile regression function and under heteroscedasticity.*

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Data Presentation and Discussion

Commodity Volatility Shocks And BRIC Sovereign Risk: A GARCH-Quantile Approach

Data Presentation and Discussion

This study is conducted with

Daily Sovereign CDS spread changes for BRIC countries (Brazil, Russia, India and China)

The 5-Year maturity CDS spread contract is used as it represents the *most traded and liquid contract*.

Logarithmic returns for the commodity and energy indices *(the Standard and Poor's Goldman Sachs Commodity Index (S&P GSCI)* and the S&P GSCI energy sub-index).

The S&P GSCI aggregate index and the energy sub-index represent the ***most widely recognized benchmark*** for the market of global commodity and energy commodity, respectively.

Data Presentation and Discussion

Market Data At A Glance

Commodity Indices	• Three Commodity Indices initially ***developed*** by ***Goldman Sachs*** and currently ***owned and published*** by ***Standard & Poor's***
The Standard & Poor's Goldman Sachs Commodity Index (S&P GSCI)	• It contains 25 commodities:
The S&P GSCI Energy Commodities Index	• 7 are energy commodities (crude oil, natural • gas, heating oil, and RBOB gasoline) - 63% weight of the aggregate S&P GSCI index,
The S&P GSCI Non-Energy Commodities Index	• the remaining 18 weight of 37% is for non-energy commodities (agriculture, livestock, industrial metals, and precious metals).

Data Presentation and Discussion

Market Data At A Glance

Appropriate Market Data

- Data are collected from *DataStream* for the period from January 4, 2010 to August 31, 2016.
- Notably, the sample period starts 6 months after the end of the US recession, as defined by the National Bureau of Economic Research (NBER).

Data Presentation and Discussion

Data At A Glance

- Commodity exports account for at least 60% of merchandise exports in Russia and Brazil. They also account for more than 6% of GDP in India and Brazil.
- In examining the exports by commodity group:
  - energy commodities are over 83% of Russia's commodity exports, followed by 43% for India.
  - Non-energy commodities are the majority of exports in Brazil and China with 86% and 77% of commodity exports, respectively.

Table 2

Commodity Exports per Country.

Source: UNCTAD (2014) http://unctad.org/en/PublicationsLibrary/suc2014d7_en.pdf. *Russian data are obtained from* http://www.imf.org/external/pubs/ft/weo/2015/02/.

	Commodity exports as % of merchandise exports	Commodity exports as % of GDP	Exports by commodity group	
			Energy	Non-energy
BRIC countries				
Brazil	65	6.8	14	86
Russia	60.5	8.1	83.1	16.9
India	47	7.6	43	57
China	8	2.5	23	77

Data Presentation and Discussion

Data At A Glance

The EIA indicates that ***China*** and ***India*** are *net oil importers*.

This suggests the sensitivity of the Indian and Chinese economies and potentially their CDS spreads to the condition and price volatility in the global energy market.

	Commodity exports as % of merchandise exports	Commodity exports as % of GDP	Exports by commodity group	
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Empirical Results

Empirical Results

- Maximum of two lags (i.e. AR(2)) in the specification of the mean equation was the optimal choice. The GARCH (1,1) outperformed the GJR-GARCH model in all cases.

- The ARCH and GARCH parameters are positive and significant at the 1% significance level in most cases. More importantly, the stationarity and positivity constraints have been respected, and the diagnostic results show that heteroscedasticity is not present in the residual series.

Table 3
Estimation of the selected GARCH-based models.

	Brazil	Russia	India	China	Commodity	Energy
Mean equation						
Constant	−0.048	−0.137	−0.005	−0.001	0.000	0.000
AR(1)	0.102***	–	0.033	–	–	
AR(2)	–	–	0.085***	–	–	–
Variance equation						
Constant	0.255***	0.983***	0.032***	0.256***	0.000*	0.000*
ARCH	0.143***	0.183***	0.070***	0.135***	0.045***	0.049***
GARCH	0.860***	0.809***	0.883***	0.846***	0.953***	0.950***
GED parameter	–	–	–	–	1.310***	1.258***
Student (DoF)	–	7	8	–	–	–
Model diagnostics						
ARCH(10)	1.286	0.781	0.349	0.642	0.878	0.748

Notes: AR(1) and AR(2) denote respectively the value of the one- and two-lagged of the dependent variable; lagged variables; Student (DoF) indicates the student-t distribution parameter; ARCH (10) represents statistic tests of the null hypothesis of no conditional heteroscedasticity up to 10 lags for the residuals; the Box–Pierce Q-squared statistic tests the null hypothesis of no autocorrelation up to order 10 for squared values.

Empirical Results

Brazil

- The *contemporaneous* effect of commodity volatility on the volatility of CDS spreads *is positive* at the 30th quantile and above, and exhibits *stronger* relation as we move higher in the quantiles.
- Concerning causality, *past values* of commodity volatility have a *negative impact* at the 30th quantile and above.
- These results confirm the relevance of commodity volatility in shaping the volatility of CDS spreads in Brazil, and are in line with the high percentage of commodity exports as % of merchandise exports (65%).

As for the coefficient of the dummy variable, it is significantly positive at the 70th quantile, suggesting that the decline in the price of (energy) commodities since mid-2014 has increased the (mean) volatility of the Brazilian sovereign risk at 70th quantile by 0.017 (x8.5).

This latter result suggests that investors and policy-makers have to worry about the effect of *declining energy commodities prices* on the *volatility of Brazilian CDS spreads* at a the 70th quantile only (medium high volatility conditions).

Given that Brazil is more of a non-energy commodities exporter: the expansion of the biofuel industry might have played a key role in intensifying the linkages between agricultural commodities and energy commodities

Empirical Results

Russia

- The contemporaneous effect of commodity volatility is positive at the 30th quantile and above, and exhibits stronger relation as we move higher in the quantiles.
- Whereas the lagged effect of commodity volatility is negative at the same quantiles.
- These results confirm the relevance of commodity volatility in shaping the volatility of CDS spreads in Russia, and are in line with the high percentage of commodity exports as % of merchandise exports (61%).

As for the coefficient of the dummy variable, it is significantly positive at the 95th quantile, suggesting that the decline in the price of (energy) commodities since mid-2014 has increased the (mean) volatility of the Russian sovereign risk at 95th quantile by 0.138/x9 (0.151/x6).

investors and policy-makers have to worry about the effect of declining energy commodities prices on the volatility of Russian CDS spreads at the extreme upper quantile of 0.95.

Empirical Results

India

- The contemporaneous and lagged effects of commodity and energy *volatilities* are *insignificant* in all quantiles.
- However, the negative and significant coefficients associated to the dummy variable at upper quantiles (0.70 up to 0.95) ***imply that the decline in the price of energy commodities has decreased the volatility of sovereign risk in India***.
- Recall that India is one of the major consumers of (energy) commodities.

Empirical Results

China

- The contemporaneous effect of commodity volatility on the volatility of the Chinese CDS spreads is significantly positive at the 70th and 90th quantiles, whereas past values of commodity volatility have a negative impact at the same upper quantiles.
- The decline in energy commodity prices has no straight effect on the *volatility* of the Chinese CDS spreads at any quantile.
- Recall that the contribution of commodity exports as % of merchandise exports is quite marginal (8%).
- As for the contribution of energy price volatility to the volatility of Chinese sovereign risk, results show that the contemporaneous and lagged effect of energy volatility are more concentrated at the extreme upper quantile of 0.95.

Empirical Results

Highlights

- The analysis on daily data from January 4, 2010 to August 31, 2016 provides evidence that the volatility relation between commodity/energy and sovereign Credit Default Swap (CDS) markets is not the same under different volatility conditions (quantiles), although energy volatility is slightly more important than commodity volatility.
- For the cases of *energy-importing* countries (India and China), the dependence is significant only under *high volatility regimes* [upper quantiles], whereas it is significant at both *medium & high volatility* regimes [middle and upper quantiles] for *commodity-exporter* (Brazil) and energy-exporter (Russia).
- In all cases, the energy index exhibits higher impacts on the volatility transmission of CDS spreads than the commodity index (Bouri et al., 2017).

Empirical Results

Highlights

- As for the impact of the mid-2014 energy price decline, it is significant only under high volatility conditions.
  - Importantly, it was significantly positive in Brazil and Russia,
  - whereas we observed a negative impact in India and no impact in China.
- These results are not consistent with the overall view that ***all*** large energy importers have benefited from the decline in energy prices through the reduction in the volatility of their sovereign risk, given that only India has experienced a decrease in the volatility of its sovereign risk. In contrast, the volatility of sovereign risk in both commodity exporter and energy exporter has increased in the period that followed the mid-2014 energy price decline.

Empirical Results

Highlights

- We added to the work of Amstad et al. (2016) by focusing on the direct role of *volatility* of commodity and energy prices, ***as part of global risk factors*** (Amstad et al., 2016), in explaining the volatility of sovereign CDS spread changes, while differentiating between BRIC energy importers and exporters.
- In fact, we showed that the sign of this effect differs between commodity export and commodity import-dependent countries.
- While Liu et al. (2016) show that *oil price volatility* can accentuate the volatility of *country risk ratings*, we revealed that ***lower energy price had reduced the volatility of sovereign risk of commodity/energy importers***. The fact that we accounted for the difference in the impact across lower, middle, and upper quantiles allowed us to extend the findings reported by Bouri et al. (2017).

Empirical Results

Highlights

We confirmed the presence of a common component of systematic risk for both oil-exporting and oil-importing large emerging countries (Amstad et al., 2016), which is the volatility of energy (commodities).

Second, we also showed the importance of domestic fundamentals, such as the level of commodity/energy dependence (Bouri et al., 2017), in shaping the volatility of sovereign risk in BRIC countries.

Third, we indicated that the mid-2014 energy price decline had affected the volatility linkages, and decreased (increased) the volatility of sovereign risk of energy/ commodity importers (exporters).

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Delighted – The End